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Evolution of Low Mass Stars:

The evolution of stars with $M \lesssim 3 M_{\odot}$ is very different from that of the high mass stars. The difference is due to the following facts:

(1) Low mass stars have no convective core. Thus material is not mixed at the center of the star, and the transition between a core of Helium and a shell of Hydrogen is continuous.

(2) Low mass stars have central densities and temperatures that are closer to the degeneracy limit. This results in very different consequences as compared with the high mass stars.

(3) Low mass stars are closer to the Hayashi line in their main sequence location (recall that the outer layers are convective in these stars).

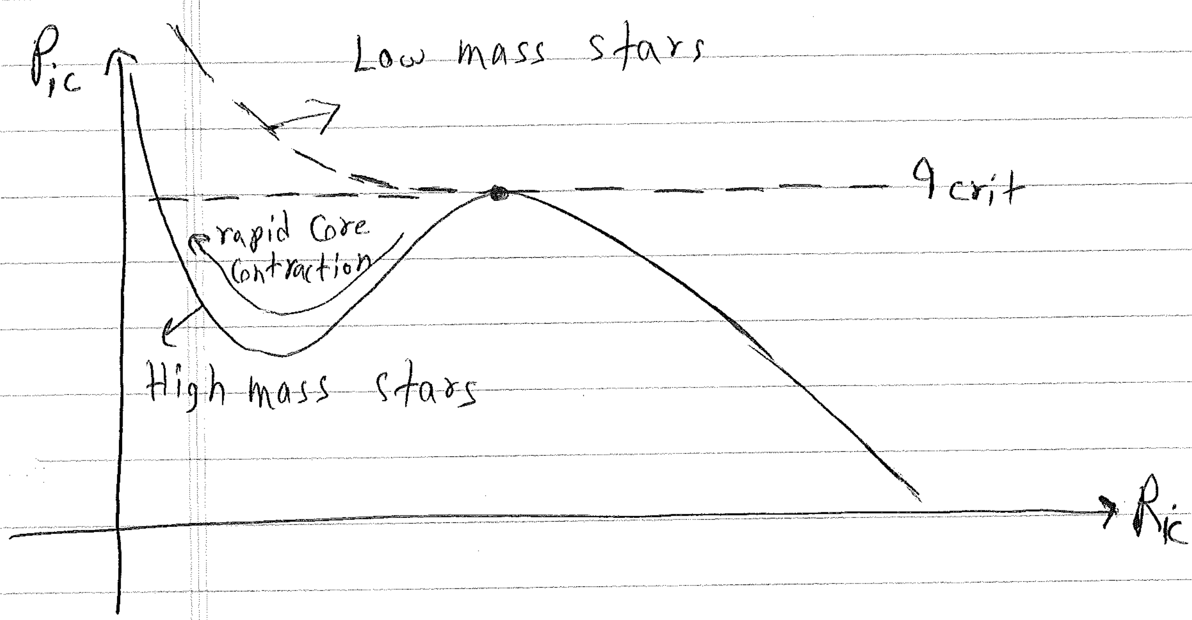
We now discuss various phases in the evolution of low mass stars.

Core and Shell Hydrogen Burning:

The main sequence again consists of gradual conversion of Hydrogen to Helium at the core. However, the low mass stars are powered by the pp-chain that has a lower temperature sensitivity than the CNO-cycle. Therefore the radius expansion (happening as a result of the change in mean molecular weight) is slower and more gradual in the low mass stars.

When Hydrogen is exhausted in the core, the core contracts and the envelope expands. The surface temperature T_e decreases, and the Hydrogen in the shell is ignited. The transition from core burning to shell burning is nearly continuous (unlike in the high mass stars) again because of the lower temperature sensitivity of the pp-chain.

Since the core is nearly degenerate, the Chandrasekhar-Shoenberg limit is fairly irrelevant for low mass stars. This can be qualitatively seen from dependence of the core pressure on the core radius shown below:



For the core masses above $\sim 0.1 M_\odot$, the core is sufficiently degenerate to circumvent the Chandrasekhar-Shoenberg limit. The core is made of degenerate isothermal Helium at this stage. Since there is no rapid core contraction, the low mass stars do not exhibit the Hertzsprung gap.

The growth of the core is slow in the initial phase, and the whole core settles to a temperature of the Hydrogen burning shell. The shell burning phase is a slow nuclear phase. For a $1 M_\odot$ star, the core and shell burning lasts for ~ 12 Gyr.

Red Giant Phase:

The contraction of the core leads to the expansion of the envelope outside the burning shell. This part of the evolution is essentially controlled by the core mass M_c. An approximate analytic description of this can be derived as follows.

As the shell burns, the core mass increases at the rate:

$$\dot{M}_c = \frac{L}{X_H E_H}$$

Here E_H is the energy released per unit mass of the Hydrogen, and X_H is Hydrogen mass fraction in the envelope. Assuming power law parameterization of the opacity and the energy production:

$\varepsilon = \varepsilon_0 \rho^{\lambda} T^{\nu}$, $\kappa = \kappa_0 \rho^{a} T^{b}$

We can use the equation of state of an ideal gas for the envelope. Under these conditions, homologous solutions will exist for the stellar structure equations. If M_c dominates over the envelope mass, $M_r \simeq M_c$ for $r > R_c$, leading to:

$$T \propto \nu \frac{M_c}{R_c} \quad , \quad L \propto \nu^{\sigma_3} M_c^{\sigma_1} R_c^{\sigma_2} \quad *$$

Here:

$$\sigma_1 = \frac{\nu(1+a) - (\lambda+1)(a+b-4)}{2+\lambda+a}$$

$$\sigma_2 = \frac{(1+a)(3-\nu) + (\lambda+1)(a+b-3)}{2+\lambda+a}$$

$$\sigma_3 = \frac{\nu(1+a) + (\lambda+1)(4-b)}{2+\lambda+a}$$

Knowing the dominant sources of opacity and energy production, we can determine σ_i. As an illustration, for the case of electron scattering ($a=b=0$) and the CNO-cycle ($\lambda=1$, $\nu \simeq 15$), we find:

$$T \propto \frac{M_c}{R_c} \quad , \quad L \propto M_c^{7.7} R_c^{-6}$$

In addition, we have $M_c \propto R_c^{-3}$ for a fully degenerate core in the non-relativistic regime. Putting the pieces together, we then have:

$$\frac{d\ln L}{d\ln M_c} = \sigma_1 + \sigma_2 \frac{d\ln R_c}{d\ln M_c}$$

To be precise, one should also take radiation pressure into account:

$$P = P_{rad} + P_{gas} = \frac{\rho k_B T}{\mu m_u} + \frac{a}{3} T^4 \quad , \quad \beta \equiv \frac{P_{gas}}{P_{gas}+P_{rad}}$$

This results in:

$$\frac{\partial \ln P}{\partial \ln \rho} = \beta \quad , \quad \frac{\partial \ln P}{\partial \ln T} = 4-3\beta$$
(approximately)

And allows us to write the equation of state as:

$P \propto \rho^{\beta} T^{4-3\beta}$

For small core masses $\beta \simeq 1$, while for large values of M_c we have $\beta \simeq 0$. In the limit of $\beta = 0$, we get $\alpha_1 = 1$ and $\alpha_2 = 0$. This results in $L \propto M_c$, and from numerical calculations we find:

$$\boxed{\frac{L}{L_\odot} = 5.92 \times 10^4 \left(\frac{M}{M_\odot} - 0.52\right)}$$

The star moves to the right and up direction in the H-R diagram in this phase and eventually moves along a Hayashi line. This happens when an outer convecte region develops and moves inward. T_e will remain nearly constant, while L increases considerably (due to expansion). When the convective region reaches the region already contaminated by the products of shell Hydrogen burning, the processed material gets well mixed and partially brought to the surface.

When the outer convective region and the Hydrogen burning shell come into contact, a discontinuity in the mean molecular weight between the Hydrogen-rich outer layer and Helium-rich layers below is produced by mixing. The mean molecular weight of the shell becomes smaller at this point. From the homology relation in equation ~~a~~ (see page 181) we see strong dependence of L on μ. A decrease in μ then results in a decrease in the luminosity. The trajectory of the star then drops back in the H-R diagram causing a transient reduction in L. After the shell source goes past the discontinuity, μ retains its lower value and L grows again with increasing M_c.

The red giant phase lasts ~ 1 Gyr for a $1 M_\odot$ star.